Formula Sheet

1 Time Value of Money

1.1 Future Value

The future value of x after n periods of growth at (annual) interest rate a compounded m times per year is

$$x(1+r)^n$$

where $r = a/m$ is the per-period interest rate.

The effective annual interest rate is

$$i = (1 + a/m)^m - 1.$$

The future value of x after t years of growth at annual growth rate d is

$$x(1+d)^t.$$

1.2 Present Value

In the following, r is the per-period discount rate, d is the annual discount rate, and there are m periods per year.

The present value of y to be received n periods later is

$$y(1+r)^{-n} = \frac{y}{(1+r)^n}.$$

The present value of y to be received t years later is

$$y(1+d)^{-t} = \frac{y}{(1+d)^t}.$$

The relationship between r and d is

$$d = (1+r)^m - 1 \quad \text{and} \quad r = (1+d)^{1/m} - 1.$$

1.3 Present Value: Perpetuities and Annuities

When the discount rate is r per period, the present value P of an annuity making n payments of C, each one period apart, starting in one period:

$$P = \frac{C}{r}(1-(1+r)^{-n}), \qquad C = \frac{Pr}{1-(1+r)^{-n}}$$

Present value of a perpetuity of C per period, starting in one period:

$$\frac{C}{r}.$$

2 Inflation

When p is a nominal cost that grows at rate h per year, the nominal cost after t years is

$$p(1+h)^t.$$

When i is an inflation rate and p is a nominal cost occurring at time u, the real cost as measured in time s dollars is

$$p(1+i)^{s-u}.$$

The real cost, as measured in base-b dollars, of an actual cost A at time t, is

$$R = A(1+f)^{b-t},$$

where f is the annual rate of inflation. If the actual cost of something at time t is A_t, and its actual cost changes at an annual rate g, then its actual cost at time u is

$$A_u = A_t(1+g)^{u-t}.$$

The relationship between the inflation rate f, the actual discount rate d_A, and the real discount rate d_R is

$$(1+f)(1+d_R) = 1+d_A.$$